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Effects of strategy-updating cost on evolutionary spatial prisoner's dilemma game

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ABSTRACT

Strategy-updating rules play fundamental roles for the persistence of cooperation in groups composed by selfish individuals. In this paper, we study the spatial evolutionary prisoner's dilemma game with the introduction of the strategy-updating cost for players, where each player is able to update its strategy if its payoffs is greater than a critical threshold. We show that there exist sudden increases of cooperation level as the temptation to defect increases for a fixed strategy-updating cost, which means a larger temptation to defect cannot always inveigle players into defection, but sometimes promote players to cooperate. This striking phenomenon is in contradiction with the previous wide cognition that a larger temptation to defect always gives rise to a lower cooperation level. This abnormal phenomenon can be explained by a systematic analysis of the payoffs earned by cooperators and defectors and the strategy-transition probabilities between cooperation and defection, respectively. In some cases, the strategy-updating cost can prevent some defectors from becoming cooperators and the increase of temptation to defect enable their payoffs to exceed the threshold of strategy-updating cost. Our results prove that the temptation to defect may have facilitation to the emergence of cooperation in the existence of strategy-updating cost, and thus provide a new understanding of the previously hidden roles of the temptation to defect for the social cooperation.

1. Introduction

Cooperation is widespread in nature and societies, the emergence of which among selfishness is a perennial problem in the evolutionary game theory as it conflicts with the theoretical basis of natural selection [1–5]. Evolutionary game theory provide a useful perspective to understand this challenging problem, which has attracted the attention of plenty of researchers, including physicists [6,7], biologists [8,9], psychologist [10,11], social sciences [12], etc. One of the most successful models in the game theory is the prisoner's dilemma game (*PDG*), which grasps the conflicts of interests between an individual and a group in the social dilemma and has become the main paradigm to understand the persistence of cooperation [13–19]. In a typical *PDG*, two involved players make choice of cooperation or defection independently [20]. If they both cooperate, they will be rewarded with R, and if they both defect, they will be penalized with P. However, if one cooperates and one defects, the defector gets temptation T and the sucker gets S. Obviously, a player tend to defect its opponent in consideration of payoff maximization.

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Due to the complexity of organization manners of human or animal society, spacial structure [21,22] and complex networks with diverse structural properties [7,23–28] have been introduced as the underling interaction structures of players. It has been found that the cooperation can persist or be prosperous by the formation of cooperator clusters to avoid the exploitation of defectors [29–39]. This successful mechanism to raise the cooperation level in the evolutionary game is referred to network reciprocity [40–43]. Network reciprocity relates diverse type of structures, such as regular lattice [21,22,44], single-layered networks [7,23], time-varying networks [45], and multilayered networks [46–49]. Along with the research of network reciprocity, some properties of network structure and the attributes of nodes have been also proven to be important for cooperation persistence, such as degree mixing [46,50], cluster [51], social diversity [20,52–55], time scale [56,57], aspirations [14,58,59] and punishment [60–64]. It is worthy of noting that the heterogeneous connectivity of players will cause the diversity of strategy-spreading ability of players [65,66], which further leads the cooperation prosperity in a network. After that, heterogeneous teaching [67] or learning activities [54,68,69] and coevolutionary rules [48,70] have been taken for important reasons for the persistence of cooperation.

In human societies [71] or biological systems [9], the change of strategy is always costly, which requires individuals to adapt themselves to new environments and access to the information about their decision scenarios. Beyond that, when one strategy is discarded and a new strategy is adopted, the resource or asset associated with the old one will sink, and new knowledge, information or resource should be acquired to match the new strategy. In the previous studies of evolutionary games, the strategy-updating rule is always designed according to the comparison of their payoffs with their aspirations [58] or these of their nearest neighbors [22,72]. However, the strategy-updating cost has not been taken into the strategy-varying mechanism in the evolutionary process. Motivated by this, we introduce the strategy-update cost in the spatial game model, where only players whose payoffs exceed the minimum threshold can update their strategies. The minimum threshold of payoff denotes the total cost in the strategy updating process. Within a very simple model, we observe the cooperators can persist under conditions with a very large temptation to defect, and we observe the sudden increases of cooperation level with the gradual increase of temptation to defect.

2. Model

In order to introduce the strategy-updating cost, we use the standard networked *PDG* with a number of individuals located on a network as their underling interaction structure. In every time step of the evolution, each individual plays games with its nearest neighbors by using a strategy either *C* or *D*, and its payoff is determined by the strategies of its opponents. In this work, we make use of the parameter settings as in ref. [21], where the reward *R* is normalized as 1, both the punishment *P* and the payoff of a sucker *S* are set as 0, and the temptation to defect $T = b > 1$. After interacting with all its neighbors, each individual obtains an accumulated payoff, which is always used to evaluate the fitness or prosperity of the individual and referred as a key quantity in determining the strategy adoption process [23,73]. It has been widely recognized that heterogeneous networks provide a favorable condition for the emergence of cooperation through the mechanism so called network reciprocity, where cooperators will tend to occupy the hubs and earn higher accumulated payoffs through the mutual cooperation with their nearest neighbors [40–43]. Some other studies consider that the average payoff, accumulated payoff divided by the connectivity degree, instead of the accumulated one is also reasonable to weigh the fitness of a player since players should pay some costs to maintain their interaction relationship [74,75]. The rationality of this method lies in that the average payoff represents the efficiency of a strategy in earning payoff for each interaction, and individuals may more concern about the average payoff as the connectivity degrees of the individuals are unchangeable. For networks with homogeneous degree distribution, there is no essential difference between accumulated payoff and average payoff in determining the evolutionary cooperation. In this paper, we use the average payoff to perform the study of evolutionary cooperation. In the strategy-updating process, each player independently and synchronously decides whether to update the strategy depending on whether its average payoff is greater than a critical threshold value θ, which denotes the minimum cost required in the strategy-updating process. If the average payoff of a player *i* is less than θ, it gives up updating its strategy and keeps its strategy unchanged. Otherwise, the player *i* decide to update its strategy and randomly choose a player *j* in the neighborhood as a reference. Specifically, the player *i* makes a comparison between her/his own average payoff P_i with that of the reference *j*, and follows *j*'s strategy with a probability [22]

$$W_{ij} = \frac{1}{1 + exp[(P_i - P_j)/K]}, \tag{1}$$

where *K* is the noise level in the strategy-replication process. If $K \to 0$, the players in the system is perfectly rational and only imitate the strategies of players with higher average payoffs. In the opposite case $K \to \infty$, the players imitate the strategy of the reference randomly [22,30,76]. In this paper, we use a usually adopted value $K = 0.1$ to explore the effects of strategy-updating cost [30,77].

We consider our model to be a simplified representation of social dilemma with decision-making or strategy-changing cost. In the decision-making process, a player has to acquire the information about the strategy used and the payoff earned by a reference in the last round. These comes with some costs, such as time, money and so on. Another non-ignorable factor is the possible risk or sunk costs in case of strategy change. Specifically, when a new strategy is adopted, the resource or asset associated with the old one will sink, and new knowledge, information or resource should be acquired to match the new strategy, which needs to pay an additional price. In this regard, individuals have to consider these costs when updating

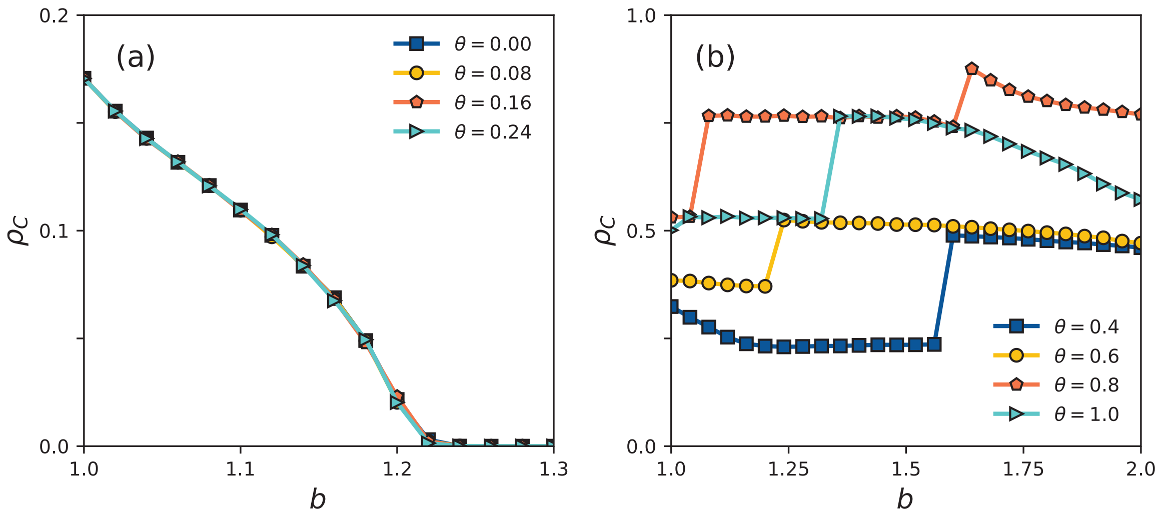

Fig. 1. (color online) Stationary cooperators density ρ_C versus the temptation to defect b. (a) shows the results for some small values of θ, i.e., $\theta = 0$, $\theta = 0.08$, $\theta = 0.16$ and $\theta = 0.24$, respectively. (b) shows the results for some large values of θ, i.e., $\theta = 0.4$, $\theta = 0.6$, $\theta = 0.8$ and $\theta = 1.0$, respectively (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.).

the strategy. Here we use the tunable parameter θ to count the total cost in the strategy-updating process, and only the individuals who can afford these costs can update their strategies. Thus, although simplified, the study of spatial *PDG* with strategy-updating cost is informative to the understanding of evolutionary cooperation among selfish individuals.

We perform the simulations for a system with a number $N = 10000$ of players sited on either a two-dimensional four-neighbor regular lattice or a scale-free network, where the players play *PDG* with their nearest neighbors. Initially, each player is assigned to a strategy either *C* or *D* randomly. After a large number steps of evolution, the system can reach a dynamic steady state in terms of the cooperator density ρ_C, which is calculated as the number of cooperators dividing the totality of players. We start the simulation from 100 different initial configurations of strategies, and each data point is the average of the last 2×10^3 time steps in the total 2×10^4.

3. Simulation and analysis

3.1. Regular lattice

Here we first show the simulation results for the two-dimensional four-neighbor regular lattice. Fig. 1 shows the impacts of the temptation to defect b on the cooperation density ρ_C for some values of strategy-updating cost θ. For some small values of θ, the cooperation level ρ_C decreases continuously with the increase of the temptation to defect b, which is entirely consistent with previous research [21,22]. However for some large values of θ, we are very surprised to observe that there exist some critical values of b_c, at which the cooperation level ρ_C can increase abruptly and discontinuously. For each value of $\theta = 0.4$, 0.6 and 1, we find there exists one abrupt jump for the cooperation level ρ_C versus b. For $\theta = 0.8$, we find there exist two abrupt jumps for the cooperation level ρ_C versus b in the parameter range $1 \le b \le 2$. This means that, the increase of the temptation to defect sometimes cannot reduce the cooperation level, but promote it for some specific strategy-updating costs.

Fig. 2 shows the relation curves of ρ_C versus θ for some values of b. It is found that the cooperation level ρ_C has a step structure: it can increase or decrease stepwise with the varying of θ, and keeps stable locally between two consecutive critical points of θ_c. This result has two implications, one of which is that a slight change of θ can lead the dramatic change of ρ_C at around the critical points of θ_c, and the other of which is that the strategy-updating cost has both positive and negative impacting on the evolutionary cooperation. When the strategy-updating cost θ exceed the maximum threshold, the cooperation level ρ_C reaches its final stable level 0.5 as all individual can not update their strategies and always keep their initial strategies in the evolutionary process.

In order to gain some insights into the abrupt change of ρ_C depending on b, we have plotted the transition probabilities $W_{C \to D}$ and $W_{D \to C}$ between cooperators and defectors as functions of the parameter b respectively in Fig. 3. When $\theta = 0.3$, it is found that the transition probabilities $W_{C \to D}$ and $W_{D \to C}$ have sudden decreases simultaneously at the critical value of $b_c = 1.2$, which leads a sudden increase of ρ_C at the same point. For $\theta = 0.7$, we can find the similar result, but the abrupt change of ρ_C occurs at a different critical value of $b_c = 1.4$. Fig. 4 shows the simulation results of $W_{C \to D}$ and $W_{D \to C}$ versus the parameter θ. We can also find that the change of θ can significantly affect $W_{C \to D}$ and $W_{D \to C}$ simultaneously, which therefore leads the tremendous changes of ρ_C.

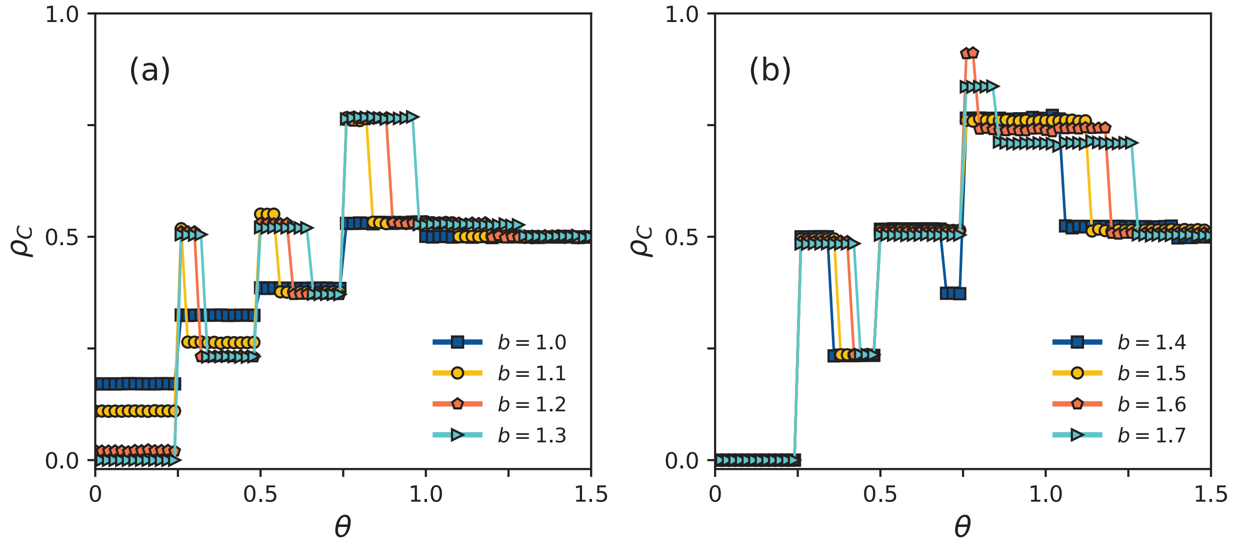

Fig. 2. (color online) Stationary cooperators density ρ_C as functions of the strategy-updating cost θ. (a) shows the results for some small values of b, i.e., $b = 1.0$, $b = 1.1$, $b = 1.2$ and $b = 1.3$, respectively. (b) shows the results for some large values of b, i.e., $b = 1.4$, $b = 1.5$, $b = 1.6$ and $b = 1.7$, respectively (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.).

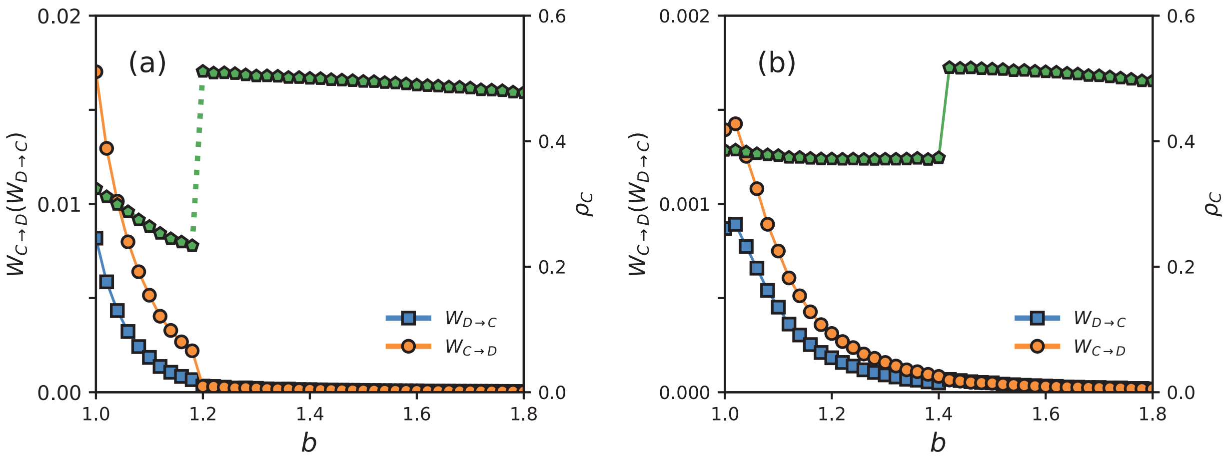

Fig. 3. (color online) The transition probability $W_{D \to C}$ from cooperators to defectors (blue squares), the transition probability $W_{C \to D}$ from defectors to cooperators (orange circles) and the cooperator density ρ_C (green pentagons) change as functions of b on four-neighbor regular lattice for $\theta = 0.3$(panel (a)) and $\theta = 0.7$ (panel (b)), respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Here we give a heuristic analysis for the emergence of the nontrivial phenomenon. For each individual, there may be a number n_C of cooperators in its neighborhood, which ranges from 0 to 4 for a four-neighbor square lattice. For a cooperator, its average payoff can be easily calculated as $P(C, n_C) = n_C/4$. While for a defector, its average payoff is $P(D, n_C) = n_C b/4$. With the increase of b, $P(D, n_C)$ increases synchronously and $P(C, n_C)$ keeps constant. When $P(D, n_C)$ reaches the critical value of θ, the transition probability $W_{D \to C}$ can have a very significant change, which gives rise to the tremendous change of ρ_C. Since n_C is in the integer range [1,4] for a four-neighbor square lattice, the critical value of b can be predicted by the equation $b_c = 4\theta/n_c$, which is very coincidence with the simulation results shown in Fig. 3.

Similarly, we can also analysis the nontrivial behavior ρ_C versus θ by the possible payoffs earned by cooperators and defectors, respectively. When $P(C, n_C)$ or $P(D, n_C)$ reach the critical value of θ_c, the transition probabilities $W_{C \to D}$ or $W_{D \to C}$ could changes greatly, which give rise to the abrupt changes of ρ_C. Specifically, when the payoffs $P(C, n_C)$ of cooperators reaches the critical value θ_c, these cooperators can change their strategy collectively and the transition probability $W_{C \to D}$ increases, which leads the sudden drop of ρ_C. However when the payoffs $P(D, n_C)$ of defectors reaches the critical value θ_c, these defectors can change their strategy collectively and the transition probability $W_{D \to C}$ increases, which leads the sudden increase of ρ_C. Therefore, the critical values of θ_c can be given by the formulas $n_C b/4$ and $n_C/4$ with the increasing

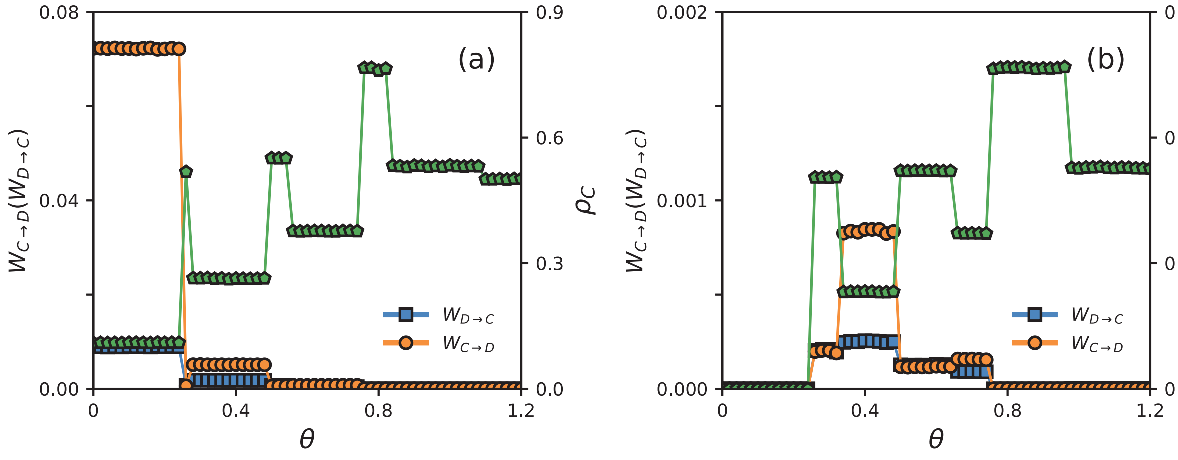

Fig. 4. (color online) The transition probability $W_{D\to C}$ from cooperators to defectors (blue squares), the transition probability $W_{C\to D}$ from defectors to cooperators (orange circles) and the cooperator density ρ_C (green pentagons) change as functions of θ on four-neighbor regular lattice for $b = 1.1$(panel (a)) and $b = 1.3$ (panel (b)), respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

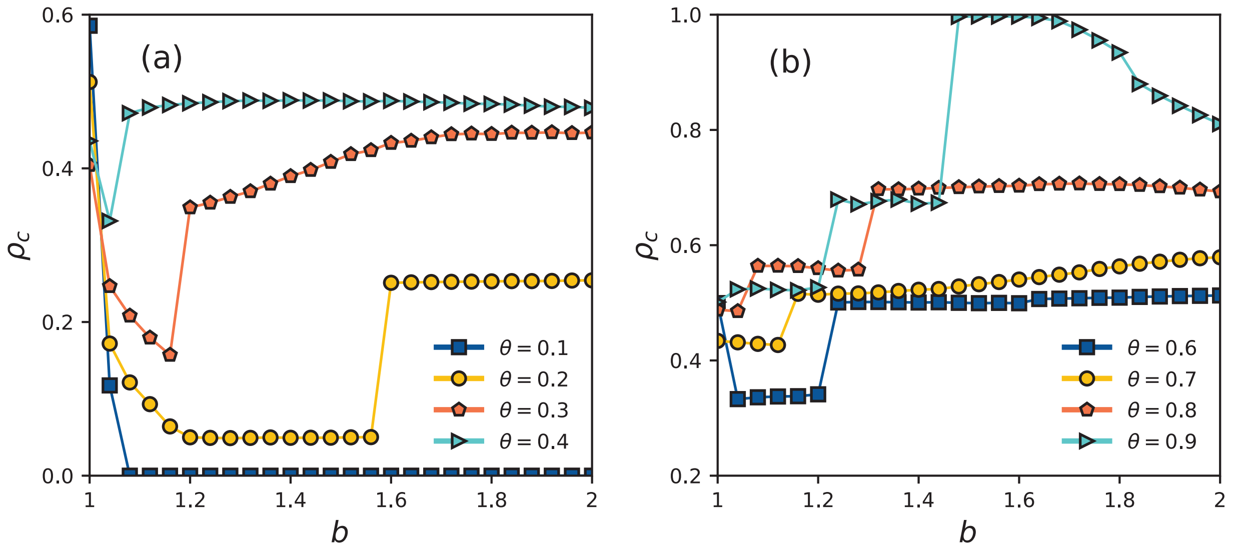

Fig. 5. (color online) Stationary cooperator density ρ_C as functions of the temptation to defect b on an eight-neighbor square lattice. (a) shows the results for some small values of θ, i.e., $\theta = 0.1$, $\theta = 0.2$, $\theta = 0.3$ and $\theta = 0.4$, respectively. (b) shows the results for some large values of θ, i.e., $\theta = 0.6$, $\theta = 0.7$, $\theta = 0.8$ and $\theta = 0.9$, respectively (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.).

of n_C from 1 to 4. Fig. 4 shows the transition probabilities $W_{C\to D}$ and $W_{D\to C}$ as functions of θ, which also validates our analysis quite well.

The above analysis reveals that the coordination number of a lattice is very important for the emergence of the step changes of ρ_C. It is meaningful to validate our analysis by studying the evolutionary cooperation on the eight-neighbor square lattice. Fig. 5 shows the cooperation density ρ_C as functions of the temptation to defect b for different θ, from which one can also observe the step structure of ρ_C versus b. For the eight-neighbor square lattice, n_C can be in the integer range [1,8], and the critical value of b can be predicted by the equation $b_c = 8\theta/n_C$, which can be verified by the results shown in Fig. 5.

Fig. 6 (a) and (b) show that the cooperator density ρ_C depends on various temptation to defect b and strategy-updating cost θ for a four-neighbor square lattice and an eight-neighbor square lattice, respectively. In Fig. 6(a), we can find some straight lines $\theta = 0.25$, $\theta = 0.5$ and $\theta = 0.75$, where the cooperation level ρ_C has a abrupt change when the parameter value of b goes through these lines. Similarly, we can also find the diagonal lines $\theta = 0.25b$, $\theta = 0.5b$, $\theta = 0.75b$ and $\theta = b$, which divides the cooperation level ρ_C into different values. In Fig. 6(b), we can find similar results, where exists some straight lines $\theta = n_C/8$ and diagonal lines $\theta = n_C b/8$ that could segment the plane into different areas. All these results indicate that

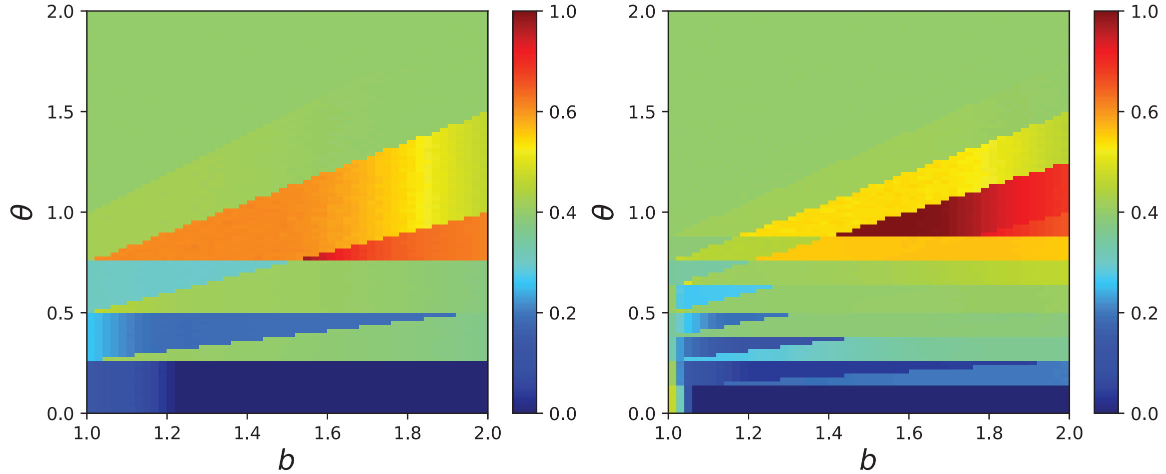

Fig. 6. (color online) The dependence of stationary cooperator density ρ_C on the temptation to defect b and strategy-updating cost θ. (a) shows the result for four-neighbor square lattice and (b) shows the result for eight-neighbor square lattice, respectively (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.).

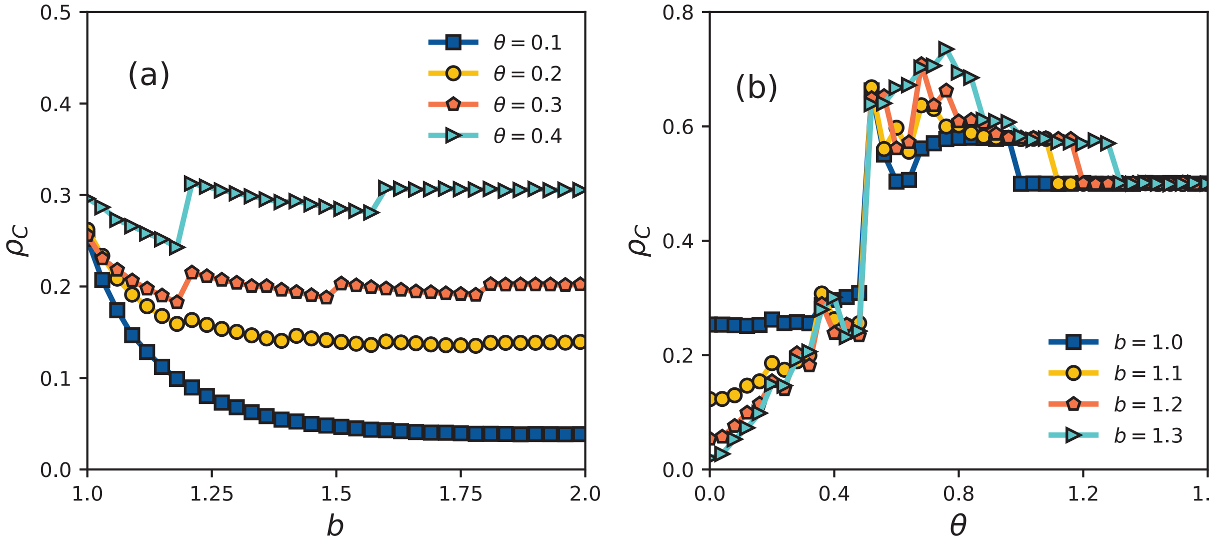

Fig. 7. (color online) The dependence of stationary cooperator density ρ_C on the temptation to defect b (a) and strategy-updating cost θ (b). In both panels, the underlying interaction structure of individuals is Barabási-Albert scale-free networks and the average connectivity is 4 (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.).

the temptation to defect could play a more complicated role than previous thought in the evolutionary cooperation when there is strategy-updating cost.

3.2. Scale-free networks

Here we study the PDG on Barabási-Albert scale-free networks (BASFN) with average connectivity 4, where players can update their strategies if their average payoffs are greater than a threshold θ. Fig. 7 (a) shows the impacts of the temptation to defect b on the cooperation density ρ_C for some values of strategy-updating cost θ. One can find that the cooperation density ρ_C decreases monotonously with the increase of the temptation to defect b if $\theta = 0$. However for some values $\theta > 0$, the phenomenon of abrupt increase of ρ_C still persist at some critical values of b_c. This means the increase of the temptation to defect can also promote cooperation level for some specific strategy-updating costs when the underlying interaction structure of individuals is of strong heterogeneity in degree distribution. Qualitative explanation for the emergence of this nontrivial phenomenon can be given by a similar way as that of regular lattice. For scale-free networks, there is a large number of nodes with low degrees. When b increases to a certain critical value, the payoffs of some defectors can gain a payoff greater that the threshold θ, and the strategy-transition probability $W_{D\to C}$ could changes greatly, which give rise to the abrupt changes of ρ_C.

Fig. 7 (b) shows the cooperation density ρ_C as functions of the strategy-updating cost θ for different values of b, from which we can also find that the strategy-updating cost θ promotes the cooperation level ρ_C firstly and then reduces the cooperation level ρ_C to a fixed level around 0.5. This result also validates that the strategy-updating cost θ can also change the transition probabilities both from C to D and from D to C dramatically and thus have significant impact to the cooperation level ρ_C.

4. Conclusion

In this paper, we have introduced the strategy-updating cost and studied its effects on the spacial evolutionary *PDG*, where the players can not update their strategies if their payoffs are less than a critical threshold θ. We found that the critical threshold of strategy-updating cost has a nontrivial effect on the persistence of cooperation on regular lattices and scale-free networks. Especially, as b increases, the cooperation level can increase suddenly at some critical values of b for some specific strategy-updating costs θ, which is opposite to the past finding that a greater temptation to defect invariably results in a smaller cooperator density. While for a fixed b, the cooperation level can decrease or increase stepwise as the strategy-updating cost θ increases, which means the strategy-updating cost θ can promote cooperation sometimes or suppress cooperation sometimes. In order for a full understanding of our results, we have analyzed the payoffs earned by C players and D players and the transition probabilities between them. When the payoffs of cooperators or defectors reach to the critical value θ of strategy-updating cost, the transition probabilities between cooperators and defectors could change greatly, which give rise to the abrupt changes of cooperation level. Our results demonstrate that the temptation to defect may promote cooperation in the presence of strategy-updating cost, thus providing new insights into the hidden role of temptation to defect in social cooperation.

The current model aim to describe the evolution of human cooperation in terms of the strategy-updating dynamic by the assumption that the players cannot update their strategies if the payoffs they earned are less than a critical threshold. Therefore less prosperous agents could not change their strategies even in case of the change could yield higher payoffs. This mechanism somehow is consistent with the phenomenon of economics that low-income individuals do not want to take risks and more likely pursuit stability [78]. However, we do concede that our current model is too simplified to represent the phenomenon in ecological system where less prosperous agents are more likely to be eliminated. In addition, we have only consider a specific *PDG* and adopt a set of parameters $R = 1,\ P = 0,\ S = 0$ and $T = b$. Actually, with the varying of the parameter settings, a 2 by 2 game can be extended to more types of games [79,80], which have been recognized as powerful tools to analysis or model the specific dilemmas in some real-world social systems [12,81]. Future work should focus on more factors that could influence the evolutionary cooperation in real social or ecological systems if there is strategy-updating cost.

Acknowledgments

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